Palestine Journal of Mathematics

Vol. 9(2)(2020) , 984–990

$*$-RICCI SOLITONS ON $(\epsilon)-$KENMOTSU MANIFOLDS

Krishnendu De, Adara M. Blaga and Uday Chand De

Communicated by Jose Luis Lopez-Bonilla

MSC 2010 Classifications: 53C15, 53C25.

Keywords and phrases: $(\epsilon)-$Kenmotsu manifolds, $*$-Ricci solitons, gradient $*$-Ricci solitons, $*$-Einstein manifold.

Abstract In the present paper we study $*$-Ricci solitons and provide the condition for a $*$-Ricci soliton in an $(\epsilon)-$Kenmotsu 3-manifold M with constant scalar curvature to be steady. Beside these, we study gradient $*$-Ricci solitons on $(\epsilon)-$Kenmotsu 3-manifolds.

1 Introduction

In modern mathematics, the methods of contact geometry play an important role. Contact geometry has evolved from the mathematical formalism of classical mechanics. The roots of contact geometry lie in differential equations as in 1872 Sophus Lie introduced the notion of contact transformation as a geometric tool to study systems of differential equations. This subject has multiple connections with the other fields of pure mathematics, and substantial applications in applied areas such as mechanics, optic, phase space of dynamical systems, thermodynamics and control theory.

The properties of a manifold depend on the nature of the metric defined on it. With the help of *indefinite metric*, A. Bejancu and K. L. Duggal [2] introduced $(\epsilon)-$*Sasakian manifolds*. Also Xufeng and Xiaoli [27] showed that *every* $(\epsilon)-$*Sasakian manifold must be a real hypersurface of some indefinite Kähler manifold.* In 2009, De and Sarkar [12] introduced the notion of $(\epsilon)-$*Kenmotsu manifolds*. Since Sasakian manifolds with indefinite metric play significant role in physics [15], our natural trend is to study various contact manifolds with indefinite metric. In 1972, K. Kenmotsu [20] introduced a new class of almost contact Riemannian manifolds which are known as *Kenmotsu manifolds*. Kenmotsu manifolds were studied by many authors such as G. Pitis [22], De and De ([9],[10],[11]), Binh, Tamassy, De and Tarafdar [3], Özgür ([23],[24]) and many others. In this paper, we introduce a new type of Ricci solitons, called $*$-*Ricci solitons in* $(\epsilon)-$*Kenmotsu manifolds* with indefinite metric which also enclose the usual case of Kenmotsu manifolds.

In 1959, Tachibana [26] introduced the notion of $*$-*Ricci tensor* on almost Hermitian manifolds. Later, in [17] Hamada studied $*$-Ricci flat real hypersurfaces in non-flat complex space forms and Blair [4] defined $*$-Ricci tensor in contact metric manifolds by

$$S^*(X,Y) = g(Q^*X,Y) = Trace\{\phi \circ R(X,\phi Y)\}, \tag{1.1}$$

where Q^* is called the $*$-*Ricci operator*.

In 1982, Hamilton [18] introduced the notion of Ricci flow to find a canonical metric on a smooth manifold. The Ricci flow is an evolution equation for metrics on a Riemannian manifold defined as follows:

$$\frac{\partial}{\partial_t} g_{ij} = -2R_{ij}. \tag{1.2}$$

Ricci solitons are special solutions of the Ricci flow equation (1.2) of the form $g_{ij} = \sigma(t)\psi_t^* g_{ij}$ with the initial condition $g_{ij}(0) = g_{ij}$, where ψ_t are diffeomorphisms of M and $\sigma(t)$ is the scaling function.

A Ricci soliton is a generalization of an Einstein metric. We recall the notion of Ricci soliton according to [6]. On a manifold M, a Ricci soliton is a triple (g, V, λ) with g a Riemannian metric, V a vector field (called potential vector field) and λ a real scalar such that

$$\pounds_V g + 2S + 2\lambda g = 0, \tag{1.3}$$

where $\pounds$ is the Lie derivative. Metrics satisfying (1.3) are interesting and useful in physics and are often referred as *quasi-Einstein* ([7],[8]).

QM
QUAESTIONES MATHEMATICAE
JOURNAL OF THE SOUTH AFRICAN MATHEMATICAL SOCIETY
44(11) 2021

Quaestiones Mathematicae

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tqma20

Almost quasi-Yamabe solitons and gradient almost quasi-Yamabe solitons in paracontact geometry

Krishnendu De & Uday Chand De

To cite this article: Krishnendu De & Uday Chand De (2021) Almost quasi-Yamabe solitons and gradient almost quasi-Yamabe solitons in paracontact geometry, Quaestiones Mathematicae, 44:11, 1429-1440, DOI: 10.2989/16073606.2020.1799882

To link to this article: https://doi.org/10.2989/16073606.2020.1799882

Published online: 25 Aug 2020.

Quaestiones Mathematicae 2021, 44(11): 1429–1440

https://doi.org/10.2989/16073606.2020.1799882

ALMOST QUASI-YAMABE SOLITONS AND GRADIENT ALMOST QUASI-YAMABE SOLITONS IN PARACONTACT GEOMETRY

Krishnendu De
Kabi Sukanta Mahavidyalaya, Bhadreswar, P.O.-Angus, Hooghly, Pin 712221, West Bengal, India.
E-Mail krishnendu.de@outlook.in

Uday Chand De
Department of Pure Mathematics, University of Calcutta, 35 Ballygunge Circular Road, Kol-700019, West Bengal, India.
E-Mail uc_de@yahoo.com

Abstract. The purpose of the present paper is to investigate the almost quasi-Yamabe soliton and gradient almost quasi-Yamabe solitons under the framework of three-dimensional normal almost paracontact metric manifolds.

Mathematics Subject Classification (2010): Primary: 53C21, 53C44, 53B30; Secondary: 53B50.

Key words: 3-dimensional paracontact manifolds, Yamabe solitons, gradient Yamabe solitons, almost quasi-Yamabe solitons.

1. Introduction. Many years ago, the fascinating topic of paracontact metric structures were introduced in [14]. After the publication of [21], paracontact metric manifolds have been investigated by multiple investigators in current years. The consequentiality of paracontact geometry, has been indicated minutely in the previous years by various articles illuminating the exchanges with the theory of para-Kähler manifolds and its appearance in pseudo-Riemannian geometry and mathematical physics ([10], [11], [9]). To know more information about paracontact metric geometry, we may refer to ([2], [14]) and references contained in those.

Several years ago, Hamilton introduced in [12] the concept of *Yamabe soliton.* Analogous to Hamilton, for a smooth vector field W and a real number λ, a semi-Riemannian metric g of a complete semi-Riemannian manifold $(M,\, g)$ is called a Yamabe soliton if it obeys

$$\frac{1}{2}\pounds_W g = (r - \lambda)g, \tag{1.1}$$

where r is the well-known scalar curvature and $\pounds$ indicates the Lie-derivative. Hither, W the vector field is termed as the soliton field of the Yamabe soliton. A Yamabe soliton is called *steady* if $\lambda = 0$ whereas *shrinking* or *expanding* according

Mathematical Sciences and Applications E-Notes
8 (2) 79-85 (2020) ©MSAEN
HTTPS://DOI.ORG/10.36753/MATHENOT.727083

A Note on Gradient $*$-Ricci Solitons

Krishnendu De*

Abstract
In the offering exposition we characterize $(k,\mu)'$- almost Kenmotsu 3-manifolds admitting gradient $*$-Ricci soliton. It is shown that in a $(k,\mu)'$- almost Kenmotsu manifold with $k<-1$ admitting a gradient $*$-Ricci soliton, either the soliton is steady or the manifold is locally isometric to a rigid gradient Ricci soliton $\mathbb{H}^2(-4)\times\mathbb{R}$.

Keywords: *$(k,\mu)'$- almost Kenmotsu manifolds, $*$-Ricci solitons, gradient $*$-Ricci solitons.*

AMS Subject Classification (2020): *Primary:* $53D15$, $53C15$; *Secondary:*$53C25$.

****Corresponding author***

1. Introduction

In the present paper we study the nullity distributions which play a functional role in contemporary mathematics. In the study of Riemannian manifolds (M,g), Gray [10] and Tanno [20] introduced the concept of k**-nullity distribution** $(k\in\mathbb{R})$, which is defined for any $p\in M$ and $k\in\mathbb{R}$ as follows:

$$N_p(k)=\{Z\in T_pM:R(X,Y)Z=k[g(Y,Z)X-g(X,Z)Y]\}, \tag{1.1}$$

for any $X,Y\in T_pM$, where T_pM denotes the tangent vector space of M at any point $p\in M$ and R denotes the Riemannian curvature tensor of type $(1,3)$. Recently, the (k,μ)**-nullity distribution** which is a generalized notion of the k-nullity distribution on a contact metric manifold $(M^{2n+1},\phi,\xi,\eta,g)$ introduced by Blair, Koufogiorgos and Papantoniou [5] and defined for any $p\in M^{2n+1}$ and $k,\mu\in\mathbb{R}$ as follows:

$$N_p(k,\mu)=\{Z\in T_pM^{2n+1}:R(X,Y)Z=k[g(Y,Z)X-g(X,Z)Y]+\mu[g(Y,Z)hX-g(X,Z)hY]\},$$

for any $X,Y\in T_pM$ and $h=\frac{1}{2}\pounds_\xi\phi$, where $\pounds$ denotes the Lie differentiation.

In 2009, Dileo and Pastore [7] introduced another generalized notion of the (k,μ)-nullity distribution which is named the $(k,\mu)'$**-nullity distribution** on an almost Kenmotsu manifold $(M^{2n+1},\phi,\xi,\eta,g)$ and is defined for any $p\in M^{2n+1}$ and $k,\mu\in\mathbb{R}$ as follows:

$$\begin{aligned}N_p(k,\mu)'=\{Z\in T_pM^{2n+1}:R(X,Y)Z &= k[g(Y,Z)X-g(X,Z)Y]\\ &+\mu[g(Y,Z)h'X-g(X,Z)h'Y]\},\end{aligned} \tag{1.1}$$

for any $X,Y\in T_pM$ and $h'=h\circ\phi$.

The idea of $*$-*Ricci tensor* on almost Hermitian manifolds was introduced by Tachibana [19] in 1959. Later, in [11] Hamada studied $*$-Ricci flat real hypersurfaces in non-flat complex space forms and Blair [4] defined $*$-Ricci tensor in contact metric manifolds by

$$S^*(X,Y)=g(Q^*X,Y)=Trace\{\phi\circ R(X,\phi Y)\}, \tag{1.2}$$

where Q^* is called the $*$-*Ricci operator*.

***Received :** 26–04–2020, **Accepted :** 10–10–2020*

FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. Vol. 35, No 4 (2020), 995–1001
https://doi.org/10.22190/FUMI2004995D

W_2-CURVATURE TENSOR ON K-CONTACT MANIFOLDS

Krishnendu De

Abstract. The object of the present paper is to obtain sufficient conditions for a K-contact manifold to be a Sasakian manifold.

Keywords: Sasakian manifold; K-contact manifold; W_2 curvature tensor.

1. Introduction

The inclination of existent mathematics is abstractions, generalizations and applications. In the offering exposition, we are entering an era of new concepts and some generalizations which play a functional role in contemporary mathematics. Contact geometry has been matured from the mathematical formalism of classical mechanics. A complete regular contact metric manifold M^{2n+1} carries a K-contact structure (ϕ, ξ, η, g), defined in terms of the almost Kähler structure (J, G) of the base manifold M^{2n}. Here the K-contact structure (ϕ, ξ, η, g) is Sasakian if and only if the base manifold (M^{2n}, J, G) is Kählerian. If (M^{2n}, J, G) is only almost Kähler, then (ϕ, ξ, η, g) is only K-contact [3]. It is to be noted that a K-contact manifold is intermediate between a contact metric manifold and a Sasakian manifold. K-contact and Sasakian manifolds have been studied by several authors such as ([2], [7], [8], [10],[18], [20],) and many others. It is well known that every Sasakian manifold is K-contact, but the converse is not true, in general. However, a three-dimensional K-contact manifold is Sasakian [9].

On the other hand, Pokhariyal and Mishra [14] have introduced new tensor fields, called W_2 and E-tensor fields, in a Riemannian manifold, and studied their relativistic properties. Then, Pokhariyal [13] and De [6] have studied some properties of this tensor fields in a Sasakian manifold and Trans-Sasakian manifold respectively.

The curvature tensor W_2 is defined by

$$W_2(X, Y, U, V) \quad = \quad R(X, Y, U, V)$$

Received May 27, 2020; accepted July 19, 2020
2020 *Mathematics Subject Classification.* 53c15, 53c25.

KYUNGPOOK Math. J. 60(2020), 821-830
https://doi.org/10.5666/KMJ.2020.60.4.821
pISSN 1225-6951 eISSN 0454-8124

Some Relativistic Properties of Lorentzian Para-Sasakian Type Spacetime

Krishnendu De
Department of Mathematics, Kabi Sukanta Mahavidyalaya, Bhadreswar, P. O. - Angus, Hooghly, Pin 712221, West Bengal, India
e-mail : krishnendu.de@outlook.in

Abstract. The object of the present paper is to classify a special type of spacetime, called Lorentzian para-Sasakian type spacetime (4-dimensional LP-Sasakian manifold with a coefficient α) satisfying certain curvature conditions.

1. Introduction

In 2002, De et.al [5] introduced the notion of Lorentzian Para-Sasakian manifolds with a coefficient α which generalizes the idea of LP-Sasakian manifolds introduced by Matsumoto [9]. Hereafter, we refer to Lorentzian Para-Sasakian manifolds with a coefficient α as LP-Sasakian manifolds with a coefficient α. Mihai and Rosca [11] also introduced the same notion of LP-Sasakian manifolds independently and obtained several results in this manifold. LP-Sasakian manifolds with a coefficient α have been studied in [3, 4]. Ikawa and his co-authors [6, 7]) investigated Sasakian manifolds with a Lorentzian metric and obtained several interesting results. Motivated by the above works, we study some relativistic properties of LP-Sasakian manifolds with a coefficient α.

The basic difference between the Riemannian and the semi-Riemannian (signature of the metric tensor g is $(+,+,+,...+,+,+)$ and $(-,-,-,...+,+,+)$ respectively) geometry is the existence of a null vector, that is, a vector v satisfying $g(v,v)=0$. A non-zero vector $v \in T_pM$ is said to be *timelike* (resp; *non-spacelike, null, spacelike*) if it satisfies $g(v,v)<0$ (resp; $\leq 0. = 0, > 0$) [13]. A Lorentzian manifold is a special case of a semi-Riemannian manifold. Spacetime means a four dimensional connected semi-Riemannian manifold (M^4, g) with Lorentz metric g of signature $(-,+,+,+)$. Here we consider a special type of spacetime which is called *Lorentzian para-Sasakian type spacetime*(4-dimensional LP-Sasakian manifold with

Received November 14, 2019; revised July 16, 2020; accepted July 21, 2020.
2010 Mathematics Subject Classification: 53B30, 53C50, 53C80.
Key words and phrases: LP-Sasakian manifold with a coefficient α, ξ-conformally flat manifold, η-Einstein manifold.

Korean J. Math. **28** (2020), No. 4, pp. 739–751
http://dx.doi.org/10.11568/kjm.2020.28.4.739

A NOTE ON ALMOST RICCI SOLITON AND GRADIENT ALMOST RICCI SOLITON ON PARA-SASAKIAN MANIFOLDS

Krishnendu De and Uday Chand De

Abstract. The object of the offering exposition is to study almost Ricci soliton and gradient almost Ricci soliton in 3-dimensional para-Sasakian manifolds. At first, it is shown that if (g, V, λ) be an almost Ricci soliton on a 3-dimensional para-Sasakian manifold M, then it reduces to a Ricci soliton and the soliton is expanding for λ=-2. Besides these, in this section, we prove that if V is pointwise collinear with ξ, then V is a constant multiple of ξ and the manifold is of constant sectional curvature -1. Moreover, it is proved that if a 3-dimensional para-Sasakian manifold admits gradient almost Ricci soliton under certain conditions then either the manifold is of constant sectional curvature -1 or it reduces to a gradient Ricci soliton. Finally, we consider an example to justify some results of our paper.

1. Introduction

A Riemannian or pseudo-Riemannian manifold (M, g) obeys a Ricci soliton equation, (see Hamilton [10]) if there exists a complete vector field V, called potential vector field satisfying

$$\frac{1}{2}\pounds_V g + S = \lambda g, \tag{1.1}$$

Received May 6, 2020. Revised November 28, 2020. Accepted November 30, 2020.
2010 Mathematics Subject Classification: 53C21, 53C25, 53C50.
Key words and phrases: 3-dimensional para-Sasakian manifold, Almost Ricci soliton, Gradient almost Ricci soliton.

SUT Journal of Mathematics
Vol. 56, No. 2 (2020), 159–169

α-almost Ricci solitons On Kenmotsu manifolds

Krishnendu De

(Received May 30, 2020)

Abstract. The current article purports to investigate α-almost Ricci solitons in the framework of Kenmotsu manifolds. Among others, we prove that an α-almost Ricci solitons on a Kenmotsu manifold is expanding. Furthermore, we take into account α-almost Ricci solitons on Kenmotsu manifolds with Codazzi type of Ricci tensor and cyclic parallel Ricci tensor. Also, Kenmotsu manifolds satisfying the curvature condition $R.R = Q(S, R)$ is studied. Ultimately, we consider an example to prove the non-existence of proper α-almost Ricci solitons on Kenmotsu manifolds and verify some results.

AMS 2010 *Mathematics Subject Classification.* 53C25, 53C35.

Key words and phrases. Kenmotsu manifold, cyclic parallel Ricci tensor, α-almost Ricci soliton, Ricci generalized pseudo-symmetric manifold.

§1. Introduction

The methods of contact geometry performed a crucial role in present-day mathematics, and therefore it became famous among the eminent researchers. Contact geometry has manifested from the mathematical formalism of classical mechanics. The essences of contact geometry lie in differential equations as in 1980, and Lie [16] publicized the concept of contact transformation as a geometric tool to study systems of differential equations. This subject has many connections with the other fields of pure mathematics. In the offering exposition, we are entering some generalizations which perform an outstanding role in coeval mathematics.

Several years ago, to find a canonical metric on a smooth manifold, Hamilton [13] revealed the idea of Ricci flow. A Ricci soliton is nothing but a generalization of an Einstein metric. We recollect the idea of Ricci soliton, according to [4]. Let us presume a manifold M endowed with a Riemannian metric g, a vector field V, called potential vector field and λ a real scalar such

International Journal of Geometric Methods in Modern Physics
Vol. 18, No. 1 (2021) 2150007 (11 pages)

DOI: 10.1142/S0219887821500079

World Scientific
www.worldscientific.com

A note on gradient solitons on para-Kenmotsu manifolds

Krishnendu De*

Department of Mathematics
Kabi Sukanta Mahavidyalaya, Bhadreswar, P.O.-Angus
Hooghly 712221, West Bengal, India
krishnendu.de@outlook.in

Uday Chand De

Department of Pure Mathematics
University of Calcutta, 35, Ballygaunge Circular Road
Kolkata 700019, West Bengal, India
uc_de@yahoo.com

Received 17 March 2020
Accepted 13 October 2020
Published 25 November 2020

The purpose of the offering exposition is to characterize gradient Yamabe, gradient Einstein and gradient m-quasi Einstein solitons within the framework of 3-dimensional para-Kenmotsu manifolds. Finally, we consider an example to prove the result obtained in previous section.

Keywords: 3-dimensional para-Kenmotsu manifolds; Yamabe solitons; gradient Yamabe solitons; m-quasi Einstein solitons.

Mathematics Subject Classification 2020: 53B30, 53B50, 53C15

1. Introduction

The Yamabe flow lies in the fact that it is an inherent geometric deformation to the metrics of constant scalar curvature which is very significant in coeval mathematics. It is pointed that Yamabe flow fits to the fast diffusion case of the plasma equation in mathematical physics. The Yamabe flow is identical to the Ricci flow (defined by $\frac{\partial}{\partial_t} g(t) = -2S(t)$, where S stands for the Ricci tensor) for dimension $n = 2$. However, in dimension $n > 2$, the Yamabe and Ricci flows do not assent, since the first one preserves the conformal class of the metric but the Ricci flow does not in general. Ricci solitons are special solutions of the Ricci flow equation of the form $g_{ij} = \sigma(t)\psi_t^* g_{ij}$ with the initial condition $g_{ij}(0) = g_{ij}$, where ψ_t are diffeomorphisms of M and $\sigma(t)$ is the scaling function, where as a Yamabe soliton (defined as the

*Corresponding author.

Honam Mathematical J. **43** (2021), No. 1, pp. 115–122
https://doi.org/10.5831/HMJ.2021.43.1.115

k-ALMOST YAMABE SOLITONS ON KENMOTSU MANIFOLDS

Krishnendu De* and Uday Chand De

Abstract. In this current article, we intend to investigate k-almost Yamabe and gradient k-almost Yamabe solitons inside the setting of three-dimensional Kenmotsu manifolds.

1. Introduction

In [11] several years ago, Hamilton publicized the concept of *Yamabe soliton.* According to the author, a Riemannian metric g of a complete Riemannian manifold (M^n, g) is called a *Yamabe soliton* if it obeys

$$\frac{1}{2}\pounds_W \; g = (r-\lambda)\; g, \tag{1}$$

where W, λ, r and $\pounds$ indicates a smooth vector field, a real number, the well-known scalar curvature and Lie-derivative respectively. Here, W is termed as the soliton field of the *Yamabe soliton.* A *Yamabe soliton* is called shrinking or expanding according as $\lambda > 0$ or $\lambda < 0$, respectively whereas *steady* if $\lambda = 0$. Yamabe solitons have been investigated by several geometers in various contexts (see, [2], [3], [10], [17], [20]). The so called *Yamabe soliton* becomes the *almost Yamabe soliton* if λ is a C^∞ function. In [1], Barbosa and Ribeiro introduced the above notion which was completely classified by Seko and Maeta in [16] on hypersurfaces in Euclidean spaces.

The Yamabe soliton reduces to a *gradient Yamabe soliton* if the soliton field W is gradient of a C^∞ function $\gamma : M^n \to \mathbb{R}$. In this occasion, from (1) we have

$$\nabla^2\gamma = (r-\lambda)g, \tag{2}$$

where $\nabla^2\gamma$ indicates the Hessian of γ. The idea of gradient Yamabe soliton was generalized by Huang and Li [12] and named as *quasi-Yamabe gradient soliton.*

Received October 23, 2020. Revised December 26, 2020. Accepted January 5, 2021.
2020 Mathematics Subject Classification. 53C25, 53D10.
Key words and phrases. Kenmotsu manifold, Yamabe solitons, k-almost Yamabe solitons.
*Corresponding author

Adv. Appl. Clifford Algebras (2021) 31:38

https://doi.org/10.1007/s00006-021-01145-4

Advances in Applied Clifford Algebras

Investigation of Generalized $\mathcal{Z}$- Recurrent Spacetimes and $f(\mathcal{R}, T)$-Gravity

Krishnendu De* and Uday Chand De

Communicated by Roldão da Rocha

Abstract. This article carries out the investigation to obtain the condition under which the generalized $\mathcal{Z}$-recurrent spacetimes (briefly, $G\mathcal{Z}_4$ spacetimes) become a perfect fluid spacetime. It is shown that in a $G\mathcal{Z}_4$ spacetime obeying Codazzi type and cyclic parallel $\mathcal{Z}$ tensor represents a perfect fluid spacetime, provided the basic vector fields are co-directional. Next, we prove that a conformally flat $G\mathcal{Z}_4$ spacetime represents the dark matter era under certain conditions. We also characterize the perfect fluid Ricci symmetric $G\mathcal{Z}_4$ spacetimes. Moreover, we investigate perfect fluid $G\mathcal{Z}_4$ spacetimes in $f(\mathcal{R}, T)$-gravity theory.

Mathematics Subject Classification. 83C20, 83F05, 83D05, 53C25.

Keywords. Generalized $\mathcal{Z}$-recurrent manifolds, Codazzi type of Ricci tensor, Cyclic parallel Ricci tensor, Perfect fluid spacetime, $f(\mathcal{R}, T)$-gravity.

1. Introduction

It is well circulated that symmetric spaces perform a significant part in differential geometry. In the general theory of relativity, symmetries also furnish the solution of Einstein's field equations. Spacetime means a four-dimensional time-oriented Lorentzian Manifold. Spacetimes are represented as curved manifolds and the gravitational field is the curvature of spacetimes and the energy-momentum tensor is its root. The geometrical and physical properties of several spacetimes have been exhaustively presented in ([1], [25], [27], [28], [45]). In the late twenties, Cartan [9] has started the investigation of Riemannian symmetric spaces. Specifically, he got a classification of those spaces.

*Corresponding author.

Published online: 17 May 2021

Birkhäuser